Introduction

Rock masses may present anisotropy in their shear strength, due to the presence of one or more plane of weakness. External or internal loads (due to pressure or temperature change or mass removal) may change the stress state acting on the weak plane. Consequently, plastic failure may happen even under compressive load.

The failure may happen in two modes: a sliding failure on the weak plane or an intrinsic failure of the rock mass. The rock matrix is expected to behave elastically or at the worst in a brittle way, being represented by a non-associated Mohr-Coulomb behavior, while the sliding failure is represented by the evaluation of Mohr-Coulomb criteria as the on an explicitly defined plane

The original approach developed by Jaeger (1960) is here implemented and tested in OpenGeoSys, to reproduce the uniaxial compressive strength dependency on loading direction.

The plane of weakness model is suitable when a single, well defined orientation of discontinuity is present, as noted by Brady and Brown (2013). It is rather straightforward to extend the model presented here to include 2 or more plane of weakness orientations.

1. Plasticity

The plastic behavior of the media is evaluated according to the Mohr-Coulomb criterion, both for the rock matrix and for the oriented plane of weakness. Internal cohesion c_0 and friction angle ϕ_0 characterize the maximum allowed shear stress τ for the rock matrix failure at a given normal effective stress :

$$\tau_{max} = \tan(\theta_0)\,\sigma_n' + c_0 \qquad [1]$$

While failure along the weakness plane is characterized by internal cohesion c_{pw} and friction angle ϕ_{pw}, as well as by stresses calculated on the plane of weakness (shear τ_{pw} and normal σ_{pw_n}:

$$\tau_{pw_{max}} = \tan\left(\theta_{pw}\right)\sigma_{pw_n} + c_{pw} \qquad [2]$$

More in detail, the Given a stress tensor σ_{ij} expressed in a Cartesian reference system xyz, the components are rotated to the local reference system $x'y'z'$, where the z' direction is the direction normal to the plane and the x' and y' are the in-plane directions. The rotation matrix $\boldsymbol{A}$ to express the stress tensor on the reference system of the plane of weakness having normal (n_x,n_y,n_z) is performed by assembling the rotation matrix (for $n_z < 1$):

$$\boldsymbol{A} = \begin{bmatrix} -\frac{n_y}{\sqrt[2]{1-n_z^2}} & -\frac{n_x}{\sqrt[2]{1-n_z^2}} & 0 \\ -\frac{n_x n_z}{\sqrt[2]{1-n_z^2}} & \frac{n_y n_z}{\sqrt[2]{1-n_z^2}} & -\sqrt[2]{1-n_z^2} \\ n_x & -n_y & -n_z \end{bmatrix} \quad [3]$$

If n_z=1, all the elements are null, apart from the bottom right element being 1. The stress tensor in the plane of weakness reference system can be calculated as

$$\bar{\bar{\sigma}}_{\prime} = \boldsymbol{A}\bar{\bar{\sigma}}\boldsymbol{A}^T \text{ , or component-wise } \sigma_{i'j'} = a_{i'k}a_{j'l}\sigma_{kl} \quad [4]$$

The effective normal component σ_n (negative if compressional) and the shear component τ will respectively be:

$$\sigma_n = \sigma_{z'z'} + p \quad [5]$$

$$\tau = \sqrt[2]{\sigma_{x'z'}^2 + \sigma_{y'z'}^2} \quad [6]$$

Where p represents the scalar field pore pressure: it is invariant for rotation and it does not affect the shear stress magnitude.

The returning mapping algorithm is used for the non-associated elasto-plasticity model. The yield function F is defined as:

$$F = \tau + \sigma_n' \tan(\varphi_j) - c_{0_j} \quad [7]$$

where φ_j and c_{0j} define respectively the friction angle and the cohesion of the plane of weakness.

To define the shear plastic flow the potential function G is used, with ω_j being the dilation angle of the plane of weakness:

$$G = \tau + \sigma_n \tan(\omega_j) \quad [8]$$

The trial stress increment is calculated as pure elastic stress increment, considering the strain tensor $\bar{\bar{\varepsilon}}$ and the fourth-order stiffness tensor $\boldsymbol{E}$.

$$\Delta\sigma_{ij} = \boldsymbol{E}_{ijkl}(\Delta\varepsilon_{kl}) \quad [9]$$

If the yield function F with the trial stress is positive, the stress will have be recalculated taking into account a plastic correction:

$$\Delta\sigma_{ij} = E_{ijkl}\left(\Delta\varepsilon_{kl} - \Delta\lambda \,{}^{\partial G}/_{\partial\sigma_{ij}}\right) \quad [10]$$

Where $\Delta\lambda$ is the term resulting from the interplay between the shear plastic flow function and the yield function:

$$\Delta\lambda = \frac{\left(\partial F/\partial\sigma_{ij}\right) E_{ijkl}\Delta\varepsilon_{kl}}{\left(\partial F/\partial\sigma_{ij}\right) E_{ijkl}\left(\partial G/\partial\sigma_{kl}\right)} \qquad [11]$$

The resulting components of stress can be then expressed in the local coordinate system and back transformed to the initial Cartesian reference system xyz:

$$\bar{\bar{\sigma}} = \boldsymbol{A}^T \bar{\bar{\sigma}}_{\prime} \boldsymbol{A} \qquad [12]$$

2. Verification

Assuming a 2-d stress state determined by the principal stresses σ_1 and σ_2, where the plane of weakness is at an angle β with the major principal stress, rupture takes place if σ_1 reaches a certain critical value σ_C. Jaeger et al. (2009) found a closed form solution to express the critical value σ_C for the rock matrix:

$$\sigma_c = N_\theta \sigma_2 - 2c_0\sqrt{N_\theta} \qquad [13]$$

Where $N_\theta = \frac{1+\sin(\theta_0)}{1-sin(\theta_0)}$. Slip failure will happen on the plane of weakness for a critical value σ_C:

$$\sigma_c = \sigma_2 - \frac{2\left(c_{pw} - \sigma_2 \tan\theta_{pw}\right)}{\left(1 - \tan\theta_{pw}\tan\beta\right)\sin 2\beta} \qquad [14]$$

For a given stress state and a given plane of weakness orientation, rupture would happen in the rock matrix or along the plane of weakness according to which of the σ_C calculated in [13] and [14] is the smallest.

To test the validity of the implementation, we numerically reproduce a set of uniaxial (i.e. σ_2=0) drained compressive test performed on a cylindrical sample with varying dip of the plane of weakness, comparing the critical stress value σ_c (which we can identify then as the uniaxial compressive strength UCS of the sample) as computed by [13] and [14] and the UCS numerically obtained. In Table 1 the parameters characterizing the synthetic rock sample are noted, while in Figure 1 the analytical and the numerical UCS values are compared.

The tests are assumed to happen in fully drained conditions, during compression of the sample there is no pressure build-up and therefore stress condition are homogeneous across the sample.

Table 1. Parameter of the synthetic rock sample to test UCS

Parameter (unit)	Matrix	Plane of weakness
Bulk modulus (MPa)	100	---

Shear modulus (MPa)	70	---
Cohesion (kPa)	2	1
Friction angle (°)	40	30
Tensile strength (kPa)	0.5	0.5

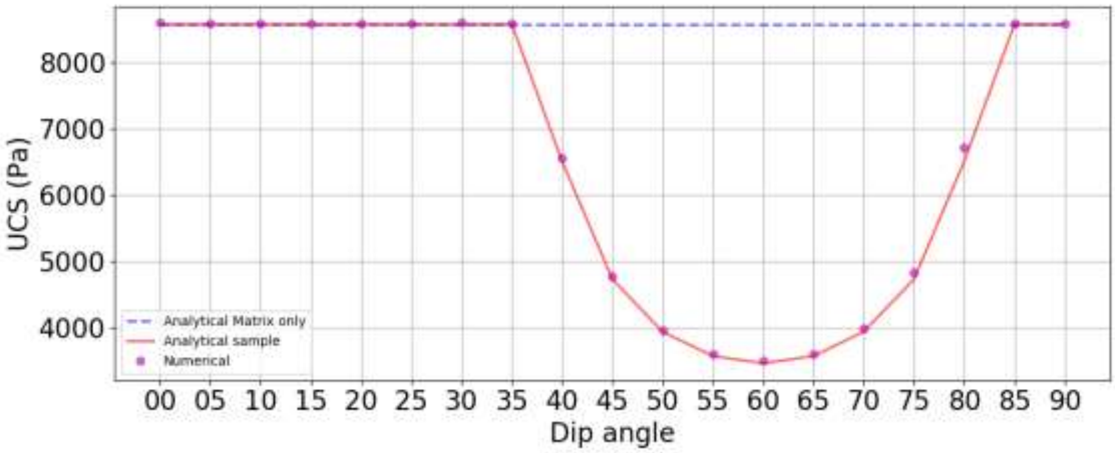

Figure 1. Resulting UCS from numerical simulations (dots) and comparison with analytical solution: the failure may happen along the weakness plane only for certain dip angles, otherwise the UCS along the plane of weakness [14] is larger than the UCS of the matrix [13]

The numerical results shows good agreement with respect to the theory, both in predicting the variation in UCS value and on the failure character, discriminating as expected between plastic behavior along the plane of weakness or due to the rock matrix.

3. Syntax

In the .msp file, the following block for a material presenting a plane of weakness must be included

```
$PLASTICITY                    //FOR THE MATRIX material
  MOHR-COULOMB
    2.e3                       //cohesion
    40                         //friction angle
    0.0                        //dilation angle
    1e9                        //tensile strength
    0                          //curve number for strain hardening of cohesion
    0                          //curve number for strain hardening of friction angle
 $WEAKNESS_PLANE                    //FOR THE PLANE OF WEAKNESS (pow)
MICRO_STRUCTURE_TENSOR 1 1 1        //needed for stress rotation, leave it like this
WEAKPLANE_NORM nx ny nz             //normal vector to the plane of weakness in OGS coordinates
    1.e3              // pow cohesion
    40                              //pow friction angle
    0.0                             //pow dilation angle
    1e9                             //pow tensile strength (tensile failure not implemented yet)
    0                               //pow curve number for strain hardening of cohesion
    0                               //pow curve number for strain hardening of friction angle
```

4. Benchmark

In the benchmark zip file, there are included the input files and the python script to reproduce the Figure 1. Run every simulation, then launch the script *compare_num-analyt.py*

References

Brady, B. H. G., & Brown, E. T. (2006). *Rock Mechanics: For underground mining* (3rd ed.). https://doi.org/10.1007/978-1-4020-2116-9

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